ENTERING ALGEBRA 2 SUMMER PACKET

Skills Handbook

Percents and Percent Applications

Percent means "per hundred." Find fraction, decimal, and percent equivalents by replacing one symbol for *hundredths* with another.

Example 1

Write each number as a percent.

a. $0.082 = 8.2\%$

Move the decimal point two places to the right and write a percent sign.

b. $\frac{3}{5} = \frac{60}{100} = 60\%$

Write the fraction as hundredths. Then replace the hundredths with a percent sign.

c. $1\frac{1}{6} = \frac{7}{6} = 1.166\overline{6} = 116.\overline{6}\%$

First, use $7 \div 6$ to write $1\frac{1}{6}$ as a decimal.

Example 2

Write each percent as a decimal.

a. $50\% = 0.50 = 0.5$

b. $\frac{1}{2}\% = 0.5\% = 00.5\% = 0.005$

Move the decimal point two places to the left and drop the percent sign.

Example 3

Use an equation to solve each percent problem.

a. What is 30% of 12?

$n = 0.3 \times 12$

$n = 3.6$

b. 18 is 0.3% of what?

$18 = 0.003 \times n$

$\frac{18}{0.003} = \frac{0.003n}{0.003}$

$6000 = n$

c. What percent of 60 is 9?

$n \times 60 = 9$

$60n = 9$

$n = \frac{9}{60} = 0.15 = 15\%$

Exercises

Write each decimal as a percent and each percent as a decimal.

1. 0.46 **2.** 1.506 **3.** 0.007 **4.** 8% **5.** 103.5% **6.** 3.3%

Write each fraction or mixed number as a percent.

7. $\frac{1}{4}$ **8.** $\frac{3}{8}$ **9.** $\frac{2}{3}$ **10.** $\frac{4}{9}$ **11.** $1\frac{3}{20}$ **12.** $\frac{1}{200}$

Use an equation to solve each percent problem. Round your answer to the nearest tenth, if necessary.

13. What is 25% of 50?

14. What percent of 58 is 37?

15. 120% of what is 90?

16. 8 is what percent of 40?

17. 15 is 75% of what?

18. 80% of 58 is what?

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Operations With Fractions

To add or subtract fractions, use a common denominator. The common denominator is the least common multiple of the denominators.

Example 1

Simplify $\frac{2}{3} + \frac{3}{5}$.

$\frac{2}{3} + \frac{3}{5} = \frac{2}{3} \cdot \frac{5}{5} + \frac{3}{5} \cdot \frac{3}{3}$ For 3 and 5, the least common multiple is 15.

$= \frac{10}{15} + \frac{9}{15}$ Write $\frac{2}{3}$ and $\frac{3}{5}$ as equivalent fractions with denominators of 15.

$= \frac{19}{15}$ or $1\frac{4}{15}$ Add the numerators.

Example 2

Simplify $5\frac{1}{4} - 3\frac{2}{3}$.

$5\frac{1}{4} - 3\frac{2}{3} = 5\frac{3}{12} - 3\frac{8}{12}$ Write equivalent fractions.

$= 4\frac{15}{12} - 3\frac{8}{12}$ Write $5\frac{3}{12}$ as $4\frac{15}{12}$ so you can subtract the fractions.

$= 1\frac{7}{12}$ Subtract the fractions. Then subtract the whole numbers.

To multiply fractions, multiply the numerators and multiply the denominators. You can simplify by using a greatest common factor.

Example 3

Simplify $\frac{3}{4} \cdot \frac{8}{11}$

Method 1 $\frac{3}{4} \cdot \frac{8}{11} = \frac{24}{44} = \frac{24 \div 4}{44 \div 4} = \frac{6}{11}$

Divide 24 and 44 by 4, their greatest common factor.

Method 2 $\frac{3}{\cancel{4}} \cdot \frac{\cancel{8}^{2}}{11} = \frac{6}{11}$

Divide 4 and 8 by 4, their greatest common factor.

To divide fractions, use a reciprocal to change the problem to multiplication.

Example 4

Simplify $3\frac{1}{5} \div 1\frac{1}{2}$

$3\frac{1}{5} \div 1\frac{1}{2} = \frac{16}{5} \div \frac{3}{2}$ Write mixed numbers as improper fractions.

$= \frac{16}{5} \cdot \frac{2}{3}$ Multiply by the reciprocal of the divisor.

$= \frac{32}{15}$ or $2\frac{2}{15}$ Simplify.

Exercises

Perform the indicated operation.

1. $\frac{3}{5} + \frac{4}{5}$	**2.** $\frac{1}{2} + \frac{2}{3}$	**3.** $4\frac{1}{2} + 2\frac{1}{3}$	**4.** $5\frac{3}{4} + 4\frac{2}{5}$	**5.** $\frac{2}{3} - \frac{3}{7}$
6. $5\frac{1}{2} - 3\frac{2}{5}$	**7.** $7\frac{3}{4} - 4\frac{4}{5}$	**8.** $3\frac{4}{5} \cdot 10$	**9.** $2\frac{1}{2} \cdot 3\frac{1}{5}$	**10.** $6\frac{3}{4} \cdot 5\frac{2}{3}$
11. $\frac{1}{2} \div \frac{1}{3}$	**12.** $\frac{6}{5} \div \frac{3}{5}$	**13.** $8\frac{1}{2} \div 4\frac{1}{4}$	**14.** $\frac{8}{9} - \frac{2}{3}$	**15.** $5\frac{1}{4} \cdot 8$

Ratios and Proportions

A *ratio* is a comparison of two quantities by division. You can write *equal ratios* by multiplying or dividing each quantity by the same nonzero number.

Ways to Write a Ratio

$a : b \quad a$ to $b \quad \frac{a}{b}\ (b \neq 0)$

Example 1

Write $3\frac{1}{3} : \frac{1}{2}$ as a ratio in simplest form.

$$3\tfrac{1}{3} : \tfrac{1}{2} \rightarrow \frac{3\frac{1}{3}}{\frac{1}{2}} \overset{\times 6}{\underset{\times 6}{=}} \frac{20}{3} \text{ or } 20 : 3$$

In simplest form, both terms should be integers. Multiply by the common denominator, 6.

A rate is a ratio that compares different types of quantities. In simplest form for a rate, the second quantity is one unit.

Example 2

Write 247 mi in 5.2 h as a rate in simplest form.

$$\frac{247 \text{ mi}}{5.2 \text{ h}} \overset{\div 5.2}{\underset{\div 5.2}{=}} \frac{47.5 \text{ mi}}{1 \text{ h}} \text{ or } 47.5 \text{ mi/h}$$

Divide by 5.2 to make the second quantity one unit.

A proportion is a statement that two ratios are equal. You can find a missing term in a proportion by using the cross products.

Cross Products of a Proportion

$\frac{a}{b} = \frac{c}{d} \rightarrow ad = bc$

Example 3

The Copy Center charges $2.52 for 63 copies. At that rate, how much will the Copy Center charge for 140 copies?

cost → / copies → $\quad \frac{2.52}{63} = \frac{c}{140}$	Set up a proportion.
$2.52 \cdot 140 = 63c$	Use cross products.
$c = \frac{2.52 \cdot 140}{63}$	Solve for c.
$= 5.6$ or \$5.60	

Exercises

Write each ratio or rate in simplest form.

1. 15 to 20 **2.** 85 : 34 **3.** 38 g in 4 oz **4.** 375 mi in 4.3 h **5.** $\frac{84}{30}$

Solve each proportion. Round your answer to the nearest tenth, if necessary.

6. $\frac{a}{5} = \frac{12}{15}$ **7.** $\frac{21}{12} = \frac{14}{x}$ **8.** $8 : 15 = n : 25$ **9.** $2.4 : c = 4 : 3$ **10.** $\frac{17}{8} = \frac{n}{20}$

11. $\frac{13}{n} = \frac{20}{3}$ **12.** $5 : 7 = y : 5$ **13.** $\frac{0.4}{3.5} = \frac{5.2}{x}$ **14.** $\frac{4}{x} = \frac{7}{6}$ **15.** $4 : n = n : 9$

16. A canary's heart beats 130 times in 12 s. Use a proportion to find about how many times its heart beats in 50 s.

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Simplifying Expressions With Integers

Skills Handbook

To add two numbers with the same sign, *add* their absolute values. The sum has the same sign as the numbers. To add two numbers with different signs, find the *difference* between their absolute values. The sum has the same sign as the number with the greater absolute value.

Example 1

Add.

a. $-8 + (-5) = -13$ **b.** $-8 + 5 = -3$ **c.** $8 + (-5) = 3$

To subtract a number, add its opposite.

Example 2

Subtract.

a. $4 - 7 = 4 + (-7)$
$= -3$

b. $-4 - (-7) = -4 + 7$
$= 3$

c. $-4 - 7 = -4 + (-7)$
$= -11$

The product or quotient of two numbers with the same sign is positive. The product or quotient of two numbers with different signs is negative.

Example 3

Multiply or divide.

a. $(-3)(-5) = 15$ **b.** $-35 \div 7 = -5$ **c.** $24 \div (-6) = -4$

Example 4

Simplify $2^2 - 3(4 - 6) - 12$.

$$\begin{aligned} 2^2 - 3(4 - 6) - 12 &= 2^2 - 3(-2) - 12 \\ &= 4 - 3(-2) - 12 \\ &= 4 - (-6) - 12 \\ &= 4 + 6 - 12 = -2 \end{aligned}$$

Order of Operations

1. Perform any operation(s) inside grouping symbols.
2. Simplify any terms with exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Exercises

Simplify each expression.

1. $-4 + 5$ **2.** $12 - 12$ **3.** $-15 + (-23)$ **4.** $4 - 17$ **5.** $-5 - 12$

6. $3 - (-5)$ **7.** $-8 - (-12)$ **8.** $-19 + 5$ **9.** $(-7)(-4)$ **10.** $-120 \div 30$

11. $(-3)(4)$ **12.** $75 \div (-3)$ **13.** $(-6)(15)$ **14.** $(18)(-4)$ **15.** $-84 \div (-7)$

16. $-2(1 + 5) + (-3)(2)$ **17.** $-4(-2 - 5) + 3(1 - 4)$ **18.** $20 - (3)(12) + 4^2$

19. $\frac{-15}{-5} - \frac{36}{-12} + \frac{-12}{-4}$ **20.** $5^2 - 6(5 - 9)$ **21.** $(-3 + 2^3)(4 + \frac{-42}{7})$

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Skills **Handbook**

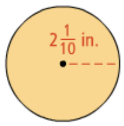
Area and Volume

The *area* of a plane figure is the number of square units contained in the figure.
The *volume* of a space figure is the number of cubic units contained in the figure.
Formulas for area and volume are listed on page 693.

Example 1

Find the area of each figure.

a.

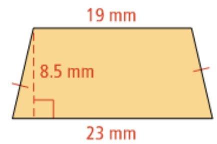


$A = \pi r^2$

$\approx \frac{22}{7} \cdot \left(\frac{21}{10}\right)^2$

$= \frac{693}{50} = 13\frac{43}{50}$ in.^{2}

b.



$A = \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}(19 + 23) \cdot 8.5$

$= 178.5$ mm^2

Example 2

Find the volume of each figure.

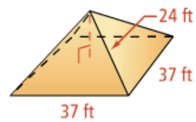
a.

$V = \frac{4}{3}\pi r^3$

$\approx \frac{4}{3} \cdot 3.14 \cdot 2.7^3$

$= 82.40616 \approx 82.4m^3$

b.



$V = \frac{1}{3}Bh$

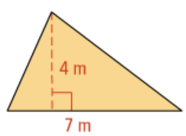
$= \frac{1}{3}(37^2) \cdot 24$

$= 10{,}952$ ft^3

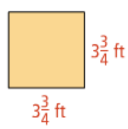
Exercises

Find the exact area of each figure.

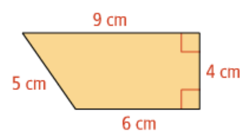
1.



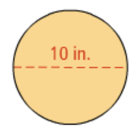
2.



3.

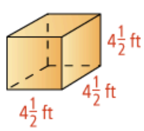


4.



Find the exact volume of each figure.

5.



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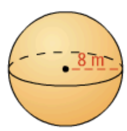

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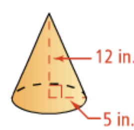

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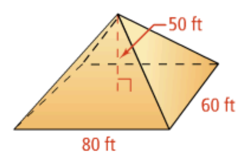

9. Find the area of a triangle with a base of 17 in. and a height of 13 in.

10. Find the volume of a rectangular box 64 cm long, 48 cm wide, and 58 cm high.

11. Find the surface area of the cube in Exercise 5.

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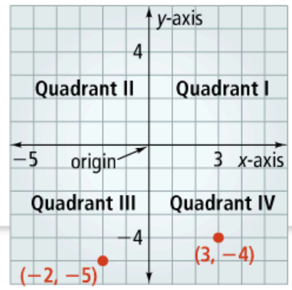
The Coordinate Plane, Slope, and Midpoint

The *coordinate plane* is formed when two perpendicular number lines intersect at a point called the origin, forming four quadrants.



Example 1

In which quadrant would you find each point?

a. (3, −4) Move 3 units right and 4 units down. The point is in Quadrant IV.

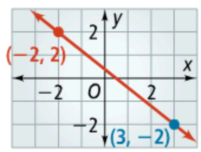
b. (−2, −5) Move 2 units left and 5 units down. The point is in Quadrant III.

To find the slope of a line on the coordinate plane, choose two points on the line and use the slope formula.

Example 2

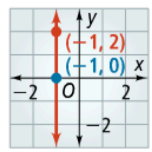
Find the slope of each line.

a.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - (-2)}{-2 - 3}$$
$$= \frac{4}{-5} \text{ or } -\frac{4}{5}$$

b.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - 0}{-1 - (-1)} = \frac{2}{0}$$

Since you cannot divide by zero, this line has an undefined slope.

If (x_m, y_m) is the midpoint of the segment joining (x_1, y_1) and (x_2, y_2), then $x_m = \frac{x_1 + x_2}{2}$ and $y_m = \frac{y_1 + y_2}{2}$.

Example 3

Find the coordinates of the midpoint of the segment with endpoints (−2, 5) and (6, −3).

$\frac{-2 + 6}{2} = 2$ and $\frac{5 + (-3)}{2} = 1$ so the midpoint is (2, 1).

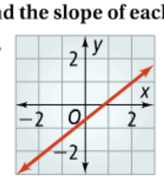
Exercises

In which quadrant would you find each point? Graph each point on a coordinate plane.

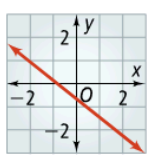
1. (3, 2) **2.** (−4, 3) **3.** (2, −3) **4.** (4, −2) **5.** (−4, −5) **6.** (−1, −3)

Find the slope of each line.

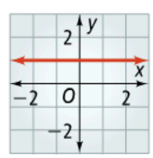
7.



8.



9.



10. the line containing (−3, 4) and (2, −6)

11. the line containing (25, 40) and (100, 55)

Find the midpoint of the segment with the given endpoints.

12. (−4, 4), (2, −5) **13.** (3, 3), (7, −6) **14.** (−1, −8), (0, −3) **15.** (3, 4), (2, −6)

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Operations With Exponents

An exponent indicates how many times a number is used as a factor.

$2^n = \blacksquare$	$10^n = \blacksquare$
$2^2 = 4$	$10^2 = 100$
$2^1 = 2$	$10^1 = 10$
$2^0 = 1$	$10^0 = 1$
$2^{-1} = \frac{1}{2}$	$10^{-1} = \frac{1}{10}$
$2^{-2} = \frac{1}{4}$	$10^{-2} = \frac{1}{100}$

Example 1

Write using exponents.

a. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

b. $a \cdot a \cdot b \cdot b \cdot b \cdot b = a^2b^4$

The patterns shown at the right indicate that $a^0 = 1$ and that $a^{-n} = \frac{1}{a^n}$.

Example 2

Write each expression so that all exponents are positive.

a. $a^{-2}b^3 = \frac{1}{a^2} \cdot b^3 = \frac{b^3}{a^2}$

b. $x^3y^0z^{-1} = x^3 \cdot 1 \cdot \frac{1}{z} = \frac{x^3}{z}$

You can simplify expressions that contain powers with the same base.

Example 3

Simplify each expression.

a. $b^5 \cdot b^3 = b^{5+3} = b^8$ Add exponents to multiply powers with the same base.

b. $\frac{x^5}{x^7} = x^{5-7} = x^{-2} = \frac{1}{x^2}$ Subtract exponents to divide powers with the same base.

You can simplify expressions that contain parentheses and exponents.

Example 4

Simplify each expression.

a. $\left(\frac{ab}{n}\right)^3 = \frac{a^3b^3}{n^3}$ Raise each factor in the parentheses to the third power.

b. $(c^2)^4 = c^{2 \cdot 4} = c^8$ Multiply exponents to raise a power to a power.

Exercises

Write each expression using exponents.

1. $x \cdot x \cdot x$ **2.** $x \cdot x \cdot x \cdot y \cdot y$ **3.** $a \cdot a \cdot a \cdot a \cdot b$ **4.** $\frac{a \cdot a \cdot a \cdot a}{b \cdot b}$

Write each expression so that all exponents are positive.

5. c^{-4} **6.** $m^{-2}n^0$ **7.** $x^5y^{-7}z^{-3}$ **8.** $ab^{-1}c^2$

Simplify each expression. Use positive exponents.

9. d^2d^6 **10.** $\frac{a^5}{a^2}$ **11.** $\frac{c^7}{c}$ **12.** $\frac{n^3}{n^6}$ **13.** $\frac{a^5b^3}{ab^8}$ **14.** $(3x)^2$

15. $\left(\frac{a}{b}\right)^4$ **16.** $\left(\frac{xz}{y}\right)^6$ **17.** $(c^3)^4$ **18.** $\left(\frac{x^2}{y^5}\right)^3$ **19.** $(u^4v^2)^3$ **20.** $(p^5)^{-2}$

21. $\frac{(2a^4)(3a^2)}{6a^3}$ **22.** $(x^{-2})^3$ **23.** $(mg^3)^{-1}$ **24.** $g^{-3}g^{-1}$ **25.** $\frac{(3a^3)^2}{18a}$ **26.** $\frac{c^3d^7}{c^{-3}d^{-1}}$

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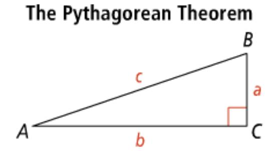
The Pythagorean Theorem and the Distance Formula

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. Use this relationship, known as the Pythagorean Theorem, to find the length of a side of a right triangle.

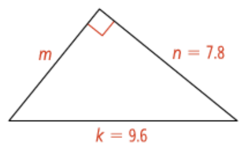
The Pythagorean Theorem



$a^2 + b^2 = c^2$

Example 1

Find m in the triangle below, to the nearest tenth.



$$m^2 + n^2 = k^2$$
$$m^2 + 7.8^2 = 9.6^2$$
$$m^2 = 9.6^2 - 7.8^2 = 31.32$$
$$m = \sqrt{31.32} \approx 5.6$$

To find the distance between two points on the coordinate plane, use the distance formula.

The distance d between any two points (x_1, y_1) and (x_2, y_2) is

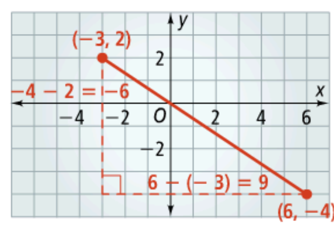
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 2

Find the distance between $(-3, 2)$ and $(6, -4)$.

$$d = \sqrt{(6 - (-3))^2 + (-4 - 2)^2}$$
$$= \sqrt{9^2 + (-6)^2}$$
$$= \sqrt{81 + 36}$$
$$= \sqrt{117}$$
$$\approx 10.8$$

Thus, d is about 10.8 units.



Exercises

In each problem, a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse. Find each missing length. Round your answer to the nearest tenth.

1. c if $a = 6$ and $b = 8$

2. a if $b = 12$ and $c = 13$

3. b if $a = 8$ and $c = 17$

4. c if $a = 10$ and $b = 3$

5. a if $b = 100$ and $c = 114$

6. b if $a = 12.0$ and $c = 30.1$

Find the distance between each pair of points, to the nearest tenth.

7. $(0, 0), (4, -3)$

8. $(-5, -5), (1, 3)$

9. $(-1, 0), (4, 12)$

10. $(-4, 2), (4, -2)$

11. $(0, 15), (17, 0)$

12. $(-8, 8), (8, 8)$

13. $(-1, 1), (1, -1)$

14. $(-2, 9), (0, 0)$

15. $(-5, 3), (4, 3)$

16. $(2, 1), (3, 4)$

17. $(3, -2), (3, 5)$

18. $(5, 4), (-3, 1)$

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Skills Handbook

Operations With Rational Expressions

A *rational expression* is an expression that can be written in the form $\frac{\text{polynominal}}{\text{polynominal}}$, where the denominator is not zero. A rational expression is in simplest form if the numerator and denominator have no common factors except 1.

Example 1

Write the expression $\frac{4x + 8}{x + 2}$ in simplest form.

$\frac{4x + 8}{x + 2} = \frac{4(x + 2)}{x + 2}$ Factor the numerator.

$= 4$ Divide out the common factor $x + 2$.

To add or subtract two rational expressions, use a common denominator.

Example 2

Simplify $\frac{x}{2y} + \frac{x}{3y}$.

$\frac{x}{2y} + \frac{x}{3y} = \frac{x}{2y} \cdot \frac{3}{3} + \frac{x}{3y} \cdot \frac{2}{2}$ The common denominator of $3y$ and $2y$ is $6y$.

$= \frac{3x}{6y} + \frac{2x}{6y}$

$= \frac{5x}{6y}$ Add the numerators.

To multiply rational expressions, first find and divide out any common factors in the numerators and the denominators. Then multiply the remaining numerators and denominators. To divide rational expressions, first use a reciprocal to change the problem to multiplication.

Example 3

Simplify $\frac{40x^2}{21} \div \frac{5x}{14}$.

$\frac{40x^2}{21} \div \frac{5x}{14} = \frac{40x^2}{21} \cdot \frac{14}{5x}$ Change dividing by $\frac{5x}{14}$ to multiplying by the reciprocal, $\frac{14}{5x}$.

$= \frac{{}^{8}\cancel{40}x^{\cancel{2}\,1}}{{}_{3}\cancel{21}} \times \frac{\cancel{14}\,{}^{2}}{\cancel{5x}\,{}_{1}}$ Divide out the common factors 5, x, and 7.

$= \frac{16x}{3}$ Multiply the numerators ($8x \cdot 2$). Multiply the denominators ($3 \cdot 1$).

Exercises

Write each expression in simplest form.

1. $\frac{4a^2b}{12ab^3}$
2. $\frac{5n + 15}{n + 3}$
3. $\frac{x - 7}{2x - 14}$
4. $\frac{28c^2(d - 3)}{35c(d - 3)}$

Perform the indicated operation.

5. $\frac{3x}{2} + \frac{5x}{2}$
6. $\frac{3x}{8} + \frac{5x}{8}$
7. $\frac{5}{h} - \frac{3}{h}$
8. $\frac{6}{11p} - \frac{9}{11p}$
9. $\frac{3x}{5} - \frac{x}{2}$
10. $\frac{13}{2x} - \frac{13}{3x}$
11. $\frac{7x}{5} + \frac{5x}{7}$
12. $\frac{5a}{b} + \frac{3a}{5b}$
13. $\frac{7x}{8} \cdot \frac{32x}{35}$
14. $\frac{3x^2}{2} \cdot \frac{6}{x}$
15. $\frac{8x^2}{5} \cdot \frac{10}{x^3}$
16. $\frac{7x}{8} \cdot \frac{64}{14x}$
17. $\frac{16}{3x} \div \frac{5}{3x}$
18. $\frac{4x}{5} \div \frac{16}{15x}$
19. $\frac{x^3}{8} \div \frac{x^2}{16}$